Matrix Algebra Practice Problems - Answers

# Part I: Computations

1. Which of the following matrices are idempotent?

Answer: A and C

1. Find the determinant of the following matrices:

Answer: **,** det(**D**) = 1

1. Find the inverse of the following matrices:
2. Consider the elementary matrices:

which multiplies row i by

which switches rows i and j

which adds times row j to row i

What are the inverses of each of these matrices? Write them in terms of the symbols above, not using an array of numbers.

Answer:

1. Find the inverse of the following matrix. Write the matrix and its inverse as products of elementary matrices.

Answer:

First we write the augmented matrix. When we do Gaussian elimination, we need to keep track of the elementary matrix corresponding to each step.

|  |  |  |
| --- | --- | --- |
| Step | Elementary Matrix | Result |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

The inverse is thus

The matrix can be written as:

using the formulas for the inverses of elementary matrices.

1. Compute the inner products of the following vectors. Which of the pairs are orthogonal?
2. , Ans: so they are orthogonal
3. , Ans: so they are not orthogonal

# Part II: Proofs

1. Prove that for square matrices , if is a right inverse of (i.e. ) and C is a left inverse of (i.e. ), then . Moreover, the inverse is unique.

Answer:

We have and . Then so . The same logic proves that this inverse is unique.

1. Prove that the product of 2 lower triangular matrices is lower triangular.

Answer:

Let be lower triangular matrices. Let and denote the row and column of , respectively. The column of is formed as a linear combination of the columns of , with the coefficients coming from the corresponding entries of column of , or . Consider an arbitrary index . Since the first entries of are zero, we have . But the columns of have zeroes in their rows. Therefore their sum must have zeroes in the rows. Thus has all zeroes in its first rows. Since was an arbitrary column, this must hold for every column, making lower triangular. Q.E.D.